# Extremal Black Holes and Strings in Linear Dilaton Vacua

Takuya Maki

Department of Physics, Tokyo Metropolitan University,
Minami-ohsawa, Hachioji-shi, Tokyo 192-03, Japan
and
Kiyoshi Shiraishi
Akita Junior College, Shimokitade-Sakura, Akita-shi, Akita 010, Japan

### Abstract

Analytic solutions to low-energy string theory, which describe arbitrary numbers of extreme 'black holes (strings)' in linear dilaton vacua, are found.

## 1 Introduction

There has recently been a great interest in the physics of Einstein equations coupled to dilaton. Dilaton gravity arises from string theory as a low-energy effective field theory and so exact solutions of the theory have been constructed from the viewpoint of underlying conformal field theories.[1] (By the way, two-dimensional dilaton gravity has also been studied intensively as a toy model in which evaporating quantum black holes are obtained.[2])

One of the most pronounced solutions to dilaton gravity closely connected to string theory is known as a linear dilaton solution.[3] Such solutions of this type can be found in general classes of string theory in arbitrary dimensions because only dilaton field varies with time in the solution.

Other bosonic gauge fields such as a vector field and an anti-symmetric tensor field are known to play important roles in construction of macroscopic, spatially-localized object in string theory [4, 10] since the dilatonic charge of the object is proportional to the charge associated with the gauge field in general, non-trivial configuration of the dilaton fields is realized around the localized object. In particular, the existence of the gauge forces admits static multisoliton solutions in string theory. Such solutions describe static configurations of an arbitrary number of maximally-charged (=extremal) black holes (or strings, membranes,...).

The present authors have presented the multi-soliton solutions for the Einstein-Maxwell-dilaton system in arbitrary dimensions.[11] In our previous works, we have only treated the limited case of electrically-charged solitons.

In the present paper, We show analytic solutions of low-energy string theory, which describes (electrically or magnetically charged) muti-'black hole' ('black string') configuration in the linear dilaton vacuum.

Actually, our solutions have singularities which are not protected by horizons. Here we call the solitonic solution 'black hole' with quotation marks above, since the special case with a single soliton corresponds to the extremal limit of a charged black hole in the limit of the vanishing cosmological constant.[6, 10, 11] We will omit the quotation marks hereafter simply for brevity.

The outline of the present paper is the following. We treat several different models and solitonic solutions in parallel sections $2 \sim 5$. In §2, we obtain the multi-magnetic black hole solutions in the low-energy string theory including a cosmological constant. The electric black hole solution with linear dilaton background is constructed in §3. The difference between electric and magnetic solutions is emphasized. Section 4 contains the description of multi-black hole solution with a non-zero three form field strength. The solution describing multi-black strings with linear dilaton is treated in §5. The final section is devoted to a brief discussion.

## 2 Magnetic black hole solution with linear dilaton in four dimensions

In this section, we consider multi-black hole solution in four dimensions. The low-energy heterotic string effective action for the bosonic fields in $D$ dimensions can be written as

$$S = \int d^D x \frac{\sqrt{-g}}{16\pi} e^{-2\phi} [R + 4(\nabla\phi)^2 - F^2 - \Lambda] , \tag{1}$$

where $\Lambda$ is a 'cosmological' constant depending on the spacetime dimensions and the central charge of a conformal field theory coupled to the string sigma model action. We must note that the 'cosmological constant' is coupled to the dilaton field $\phi$. Here we omit the term involving antisymmetric field strength as well as vector gauge field strengths except for $U(1)$ Maxwell field strength, $F$, because they are irrelevant for the classical solution we will consider in this section.

The equations of motion following from the action (1) are

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - 2F_{\mu\lambda}F_\nu{}^\lambda = 0 , \tag{2}$$

$$\partial_\mu(\sqrt{-g}e^{-2\phi}F^{\mu\nu}) = 0 , \tag{3}$$

$$R + 4\nabla^2\phi - 4(\nabla\phi)^2 - F^2 - \Lambda = 0 . \tag{4}$$

If we choose the Minkowski spacetime, a solution to (2,3,4) for $\Lambda > 0$ is

$$\phi = \phi_0(t) = \frac{\sqrt{\Lambda}}{2}t \tag{5}$$

and

$$F = 0 \,. \tag{6}$$

Here we select spatially-homogeneous evolution of the dilaton field. This classical background is known as an exact solution to string theory and called a linear dilaton solution.[3]

Now we derive analytic solutions to (2,3,4) describing an arbitrary number of magnetic black holes in the linear dilaton vacua in four dimensions. We adopt the magnetic field created by the magnetic charge $Q_i$ located at $\mathbf{x}_i$, which is written as:

$$F_{jk} = \varepsilon_{jkl} \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|^2} \frac{\mathbf{x}^l - \mathbf{x}_i^l}{|\mathbf{x} - \mathbf{x}_i|} \,, \tag{7}$$

where $\varepsilon_{jkl}$ is a totally-antisymmetric tensor $(j, k, l = 1, 2, 3)$.

For $\Lambda = 0$, the multi-black hole solution takes the form, as noted in Ref. [6]:

$$ds^2 = -dt^2 + U^4(\mathbf{x})d\mathbf{x}^2 \,, \tag{8}$$
$$e^{2\phi} = U^2(\mathbf{x}) \tag{9}$$

with

$$U^2(\mathbf{x}) = 1 + \sum_i \frac{2M_i}{|\mathbf{x} - \mathbf{x}_i|} \,, \tag{10}$$

where $M_i = |Q_i|/\sqrt{2}$.

One can easily find a time-dependent solution to (2,3,4) with $\Lambda \neq 0$ as an extension of the static one in this case. A simple solution turns out to be

$$e^{2\phi} = e^{2\phi_0(t)}U^2(\mathbf{x}) \,, \tag{11}$$

and the metric takes the same form as (8) with the same definition of $U^2$ (10). $\phi_0(t)$ is given by (5). This result exhibits a very simple mergence of linear dilaton and multi-black hole solutions.

In the next section we will investigate an electrically charged black hole solution. Electrically charged solutions may have non-trivial dependence on time because the equations of motion are not invariant under a duality rotation [4, 6] if $\Lambda \neq 0$.

## 3 Electric black hole solution with linear dilaton in arbitrary dimensions

In this section we write down analytic solutions describing electrically charged extreme black holes in a linear dilaton vacuum in $D$-dimensional low-energy

string theory. The equations of motion for metric, $U(1)$ vector field, and dilaton field are the same as (2,3,4).

For $\Lambda = 0$ and $D = 4$, the electrically-charged multisoliton solution can be derived from the magnetic solution (8,9,10) using a duality rotation.[4, 6] The electric solution is given by

$$ds^2 = -U^4(\mathbf{x})dt^2 + d\mathbf{x}^2\,, \tag{12}$$

$$e^{-2\phi} = U^2(\mathbf{x})\,, \tag{13}$$

$$A_\mu dx^\mu = U^{-2}(\mathbf{x}) \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|} dt \tag{14}$$

with

$$U^2(\mathbf{x}) = 1 + \sum_i \frac{2M_i}{|\mathbf{x} - \mathbf{x}_i|}\,, \tag{15}$$

where $M_i = |Q_i|/\sqrt{2}$.

For arbitrary $D$, the solution can be expressed as [10]

$$ds^2 = -U^4(\mathbf{x})dt^2 + d\mathbf{x}^2\,, \tag{16}$$

$$e^{-2\phi} = U^2(\mathbf{x})\,, \tag{17}$$

$$A_\mu dx^\mu = U^{-2}(\mathbf{x}) \sum_i \frac{Q_i}{(D-3)|\mathbf{x} - \mathbf{x}_i|^{D-3}} dt \tag{18}$$

with

$$U^2(\mathbf{x}) = 1 + \sum_i \frac{\mu_i}{(D-3)|\mathbf{x} - \mathbf{x}_i|^{D-3}}\,, \tag{19}$$

where $d\mathbf{x}^2 = \sum_{k=1}^{D-1} dx^k dx^k$, and $\mu_i = \sqrt{2}|Q_i|$. $(D-1)$-dimensional space is completely flat in this solution.

Now we turn to the case with non-zero $\Lambda$ in arbitrary dimension. Judging from the form of the solution with $\Lambda = 0$, we naturally take the following simplest ansatz for the metric and the dilaton field:

$$ds^2 = -U^{-4}(\mathbf{x}, t)dt^2 + d\mathbf{x}^2\,, \tag{20}$$

$$e^{-2\phi} = e^{-2\phi_0(t)}U^2(\mathbf{x}, t)\,, \tag{21}$$

where $\phi_0(t)$ is given by (5). This ansatz can automatically connect the multisoliton solution with $\Lambda = 0$ and the linear dilaton solution with Minkowski spacetime.

For the electric field, one can get the general solution to (3), which stands for the collection of the static point charges. This takes the form as

$$e^{-2\phi}U^{-2}F^{tk} = \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|^{D-2}} \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|}\,, \tag{22}$$

where $Q_i$ is the electric charge of each black hole.

We then substitute (20,21,22) into Eq. (2,3,4). $[0, i]$ components of (2) require

$$\frac{\partial^2}{\partial t \partial x^i}(e^{-2\phi_0(t)}U^2(\mathbf{x}, t)) = 0 \,. \tag{23}$$

By taking the explicit expression (22) into consideration, we find

$$U^2(\mathbf{x}, t) = 1 + \sum_i \frac{\mu_i e^{2\phi_0(t)}}{(D-3)|\mathbf{x} - \mathbf{x}_i|^{D-3}} \,, \tag{24}$$

where $\mu_i = \sqrt{2}|Q_i|$.

We find this $U^2$ satisfies the rest of Eqs. (3) and (4). Then the vector field is expressed as

$$A_t = \pm \frac{1}{\sqrt{2}}(1 - U^{-2}(\mathbf{x}, t)) \,. \tag{25}$$

Now we get the electric black hole solution with a linear dilaton.[1]

The difference from the magnetic solution is obvious. For the electric black holes. the mass of each black hole seems to increase (decrease) as the value of the linear dilaton field becomes large (small) proportionally to time. The electrically charged solution approaches the simple linear dilaton vacuum [3] either in past or in future infinity.

In the rest of this section, we investigate the multisoliton solutions with the time-dependent dilaton field in a modified model.

In order to analyze the role of dilaton, one can consider an arbitrary value for the dilaton coupling. To simplify the analysis, we first rescale the metric by defining

$$\tilde{g}_{\mu\nu} = e^{-4\phi/(D-2)}g_{\mu\nu} \,. \tag{26}$$

The action then becomes

$$\tilde{S} = \int d^D\tilde{x}\frac{\sqrt{-g}}{16\pi}\left[\tilde{R} - \frac{4}{D-2}(\tilde{\nabla}\phi)^2 - e^{-4\phi/(D-2)}\tilde{F}^2 - e^{4\phi/(D-2)}\Lambda\right] \,. \tag{27}$$

Here the action contains the standard Einstein-Hilbert action. To treat general dilaton couplings, we consider the action

$$\tilde{S} = \int d^D\tilde{x}\frac{\sqrt{-g}}{16\pi}\left[\tilde{R} - \frac{4}{D-2}(\tilde{\nabla}\phi)^2 - e^{-4a\phi/(D-2)}\tilde{F}^2 - e^{4a\phi/(D-2)}\Lambda\right] \,, \tag{28}$$

where $a$ represents dilaton couplings to other entities. Obviously, (28) is reduced to (27) if $a = 1$.

One can find analytic solutions to the field equations derived from (28) for all $a$. For non-zero values for $a$ and $\Lambda$, the set of solutions takes the form:

$$d\tilde{s}^2 = -U^{-4(D-3)/(D-3+a^2)}(\mathbf{x}, \tilde{t})d\tilde{t}^2 + (\tilde{t}/t_0)^{2/a^2}U^{4/(D-3+a^2)}(\mathbf{x}, \tilde{t})d\mathbf{x}^2 \tag{29}$$

$$e^{-4a\phi/(D-2)} = (\tilde{t}/t_0)^2 U^{4a\phi/(D-3+a^2)} \,, \tag{30}$$

$$A_\mu d\tilde{x}^\mu = \sqrt{\frac{D-2}{2(D-3+a^2)}}\frac{t_0}{\tilde{t}}(1 - U^{-2}(\mathbf{x}, \tilde{t}))d\tilde{t} \,, \tag{31}$$

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[1]We have also examined more general ansatze for $g_{\mu\nu}$ and $e^{-2\phi} \approx (f(t))^p(U^2(\mathbf{x}, t))^q$ obtained no other solution in closed form.

where

$$U^2(\mathbf{x}, \tilde{t}) = 1 + \sum_i \frac{\mu_i}{(\tilde{t}/t_0)^{D-3+a^2}(D-3)|\mathbf{x} - \mathbf{x}_i|^{D-3}} \tag{32}$$

and

$$t_0^2 = \frac{(D-2)(D-1-a^2)}{\Lambda a^4} \,. \tag{33}$$

For $\Lambda = 0$, the static solution has been shown in Ref. [10]. If we set $\tilde{t} = t_0$ in (29,30,31), these expressions coincide with those of the static solution. For $a = 0$ and $\Lambda \neq 0$, the exact solution describing extreme black holes in de Sitter background is known.[12] For $a = 0$ and $\Lambda = 0$, the solution is, of course, reduced to the well-known Papapetrou-Majumdar-Myers solution.[13]

We also find that the magnetic multisoliton solution in this modified rnodel with $\Lambda \neq 0$ can be obtained in closed form only for $a = 1$, as long as the components of metric and the exponent of the dilaton field is written in the form $\approx (f(t))^p(U^2(\mathbf{x}, t))^q$. The solution for $a = 1$ is of course reduced to the solution in §2 after the Weyl transformation and a transformation of the time coordinate.

# 4 'Magnetic' black hole solution with linear dilaton in five dimensions

In this section, we consider the multi-black hole solution in five dimensions. The low-energy string effective action for the bosonic fields in $D$ dimensions can be written as

$$S = \int d^D x \frac{\sqrt{-g}}{16\pi} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 - \frac{1}{12}H^2 - \Lambda \right], \tag{34}$$

where $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$ is an antisymmetric tensor field strength, which appears commonly in string theory.

The equations of motion derived from (34) are

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \frac{1}{4}H_{\mu\lambda\sigma}H_\nu{}^{\lambda\sigma} = 0 \,, \tag{35}$$

$$\partial_\mu(\sqrt{-g}e^{-2\phi}H^{\mu\nu\lambda}) = 0 \,, \tag{36}$$

$$R. + 4\nabla^2\phi - 4(\nabla\phi)^2 - \frac{1}{12}H^2 - \Lambda = 0 \,. \tag{37}$$

In this section, we consider the case $D = 5$, as the simplest case. The equation of motion (36) for $H$ can be satisfied by setting

$$H_{jkl} = \varepsilon_{jklm} \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|^2} \frac{x^m - x_i^m}{|\mathbf{x} - \mathbf{x}_i|} \,, \tag{38}$$

where $j, k, l, m, \cdots$ run over $1, 2, 3, 4$. This stands for an arbitrary number of charges $Q_i$ located at $\mathbf{x}_i$. The configuration is often called 'magnetic' field, for the suffices of the field strength run only over spatial indices.

The solutions to Eq. (35,36,37) with the charge configuration (38), which describe a single extremal and non-extremal black hole, have been studied in Refs. [4], [5] and [7] for various $D$.

A solution to (35,37) with (38) in five dimensions can easily be found and written in the following form:

$$ds^2 = -dt^2 + U^2(\mathbf{x})d\mathbf{x}^2 \tag{39}$$

with

$$e^{2\phi} = e^{2\phi_0(t)}U^2(\mathbf{x}) \tag{40}$$

with

$$U^2(\mathbf{x}) = 1 + \sum_i \frac{|Q_i|}{2\sqrt{2}|\mathbf{x} - \mathbf{x}_i|^2} \,, \tag{41}$$

where $\phi_0$ is given by (5).

This solution seems much alike the magnetic solution obtained in §2. The metric is independent of the value of the cosmological constant $\Lambda$. One can find similar exact solutions to the equations derived from the action which contains a $(D-2)$-form 'magnetic' field strength (instead of $H$) in $D$ dimensions. The effect of the linear dilaton enforces $\phi \to \phi + \phi_0$ in such solutions while the metric is unchanged.

# 5 Black string solution with linear dilaton in arbitrary dimensions

Black string solutions were found and investigated in Refs. [4], [8] and [9]. In particular, extremal black strings turn out to be viewed as straight fundamental strings. In this section, we exhibit multi-black string solutions in the linear dilaton vacuum in $D$ dimensions.

The equations of motion are the same ones as (35,36,37). We assume that all the strings lie along the $z$-direction, which is perpendicular to other $(D-2)$ dimensions spanned by the coordinates $x^i$. The solution is also assumed to be translationally invariant along the $z$-direction. We adopt an ansatz on the three form field strength such that

$$\sqrt{-g}e^{-2\phi}H^{tzk} = \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|^{D-3}} \frac{x^k - x_i^k}{|\mathbf{x} - \mathbf{x}_i|} , \tag{42}$$

where $Q_i$ is the 'electric' charge of each string.

By an analysis similar to the ones in the previous sections, one obtains the solution describing extremal black strings in the linear dilaton vacuum. It is given by the expression:

$$ds^2 = -U^{-2}(\mathbf{x}, t)(-dt^2 + dz^2) + d\mathbf{x}^2 \,, \tag{43}$$

$$e^{-2\phi} = e^{-2\phi_0(t)}U^2(\mathbf{x}, t) \,, \tag{44}$$

$$B_{zt} = \pm(1 - U^{-2}(\mathbf{x}, t)) \tag{45}$$

with

$$U^2(\mathbf{x}, t) = 1 + \sum_i \frac{|Q_i| e^{2\phi_0(t)}}{(D-4)|\mathbf{x} - \mathbf{x}_i|^{D-4}} \quad \text{for } D > 5 \,, \tag{46}$$

$$U^2(\mathbf{x}, t) = 1 - \sum_i |Q_i| e^{2\phi_0(t)} \ln(|\mathbf{x} - \mathbf{x}_i|) \quad \text{for } D = 4 \,. \tag{47}$$

If we send the value of $\Lambda$ to zero in this solution, we get the static configuration of multi-black strings.

We found no other analytic solution for $\Lambda \neq 0$ in modified models where the dilaton coupling takes a different value after scaling similarly to (26).

For $\Lambda = 0$, the modified model is written as, for example,

$$\tilde{S} = \int d^D \tilde{x} \frac{\sqrt{-\tilde{g}}}{16\pi} \left[ \tilde{R} - \frac{4}{D-2} (\nabla \phi)^2 - \frac{1}{12} e^{-8a\phi/(D-2)} \tilde{H}^2 \right] \,, \tag{48}$$

where $a$ represents dilaton couplings to other entities. The action (48) can be connected to the action (34) with $\Lambda = 0$ by the Weyl transformation (26), only if $a = 1$.

An analytic solution to the field equations derived from (48) for an arbitrary value for $a$ takes the form ($D > 5$):

$$d\tilde{s}^2 = U^{-2(D-4)/(D-4+2a^2)}(\mathbf{x})(-d\tilde{t}^2 + dz^2) + U^{4/(D-4+2a^2)}(\mathbf{x})d\mathbf{x}^2 \tag{49}$$

$$e^{-4a\phi/(D-2)} = U^{4a^2/(D-4+2a^2)} \,, \tag{50}$$

$$B_{zt} = \pm \sqrt{\frac{D-2}{D-4+2a^2}} (1 - U^{-2}(\mathbf{x})) \,, \tag{51}$$

where

$$U^2(\mathbf{x}) = 1 + \sum_i \frac{\mu_i}{(D-3)|\mathbf{x} - \mathbf{x}_i|^{D-3}} \,. \tag{52}$$

The static solution for $a = 1$ is equivalent to the solution (43,44,45,**??**) with $\Lambda = 0$ after the Weyl transformation and a transformation of the time coordinate are done.

# 6 Discussion

We have exhibited multisoliton solutions in string theory with a cosmological term, which represent extremal black holes (strings) in linear dilaton vacua. The metric for the electrically-charged solution has time dependence while the one of the magnetically-charged object is constant in time. The duality symmetry which exists in the theory with vanishing cosmological constant is clearly broken.

It is expected that the interactions between solitonic objects in string theory with non-zero cosmplogical constant will be explored by further study on these solutions.

Here we briefly discuss on two topics connected to string theory below.

First we investigate the tachyon in the background spacetime expressed by the solution we have obtained. The linearized equation for tachyon field $T$ is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(e^{-2\phi}\sqrt{-g}g^{\mu\nu}\partial_\nu T) + \mu^2 e^{-2\phi}T = 0\,, \tag{53}$$

where $-\mu^2$ denotes the tachyon mass. One can find that spatially homogeneous solutions for $T(t)$ are obtained in the background of magnetic black hole spacetime in four dimensions (in §2) and in five dimensions (in §4). The solution is then written as

$$T(t) \approx e^{pt}\,, \quad \text{where } p = \partial_t\phi_0 \pm \sqrt{(\partial_t\phi_0)^2 + \mu^2}\,. \tag{54}$$

This result is independent of the existence of black holes and thus is the known result: for example, in bosonic string theory, $\Lambda = (D-26)/3$ and $\mu^2 = 2$ ($\alpha' = 2$), thus we find $p = (\sqrt{D-26} \pm \sqrt{D-2})/\sqrt{12}$. For electric black hole and black string spacetime, there is no homogeneous solution of the tachyon field. This is due to the non-trivial $g_{tt}$ as well as the time-dependence of $U^2$.

The relation between the black string solution and plane-fronted waves is known.[4, 9] We briefly investigate the relation in terms of our solution with a linear dilaton. In string theory, a discrete symmetry is known for the background field. If the metric $g_{\mu\nu}$, dilaton field $\phi$, and antisymetric field $B_{\mu\nu}$ are independent of the coordinate $z$, there is a dual solution [4, 9, 14]

$$\hat{g}_{zz} = 1/g_{zz}\,, \quad \hat{g}_{z\alpha} = B_{z\alpha}/g_{zz}\,, \tag{55}$$

$$\hat{g}_{\alpha\beta} = g_{\alpha\beta} - (g_{z\alpha}g_{z\beta-B_{z\alpha}B_{z\beta}})/B_{zz}\,, \tag{56}$$

$$\hat{B}_{z\alpha} = g_{z\alpha}/g_{zz}\,, \quad \hat{B}_{\alpha\beta} = B_{\alpha\beta} - 2g_{z[\alpha}B_{\beta]z}/g_{zz}\,, \tag{57}$$

$$\hat{\phi} = \phi - \frac{1}{2}\ln g_{zz}\,, \tag{58}$$

where $\alpha, \beta$ run over all directions except $z$. Although our solution obtained in §5 is the one to low-energy field equation, the dual transformation (54) should give at least an approximation to the exact solution in string theory, which is worth studying. The dualization of the solution (43,44,45) for $D \geq 5$ yields

$$ds^2 = -\left(1 - \sum_i \frac{|Q_i|e^{2\phi_0(t)}}{(D-4)|\mathbf{x}-\mathbf{x}_i|^{D-4}}\right)dt^2 + \sum_i \frac{2|Q_i|e^{2\phi_0(t)}}{(D-4)|\mathbf{x}-\mathbf{x}_i|^{D-4}}dtdz$$

$$+ \left(1 + \sum_i \frac{|Q_i|e^{2\phi_0(t)}}{(D-4)|\mathbf{x}-\mathbf{x}_i|^{D-4}}\right)dz^2 + d\mathbf{x}^2\,, \tag{59}$$

$$\phi = \phi_0(t)\,, \quad B = 0\,. \tag{60}$$

Using new coordinates $z = (u-v)/2$ and $t = (u+v)/2$, we can express the solution as

$$ds^2 = -dudv + d\mathbf{x}^2 + \sum_i \frac{|Q_i|e^{2\phi_0((u+v)/2)}}{(D-4)|\mathbf{x}-\mathbf{x}_i|^{D-4}}du^2 \tag{61}$$

with a dilaton field $\phi_0((u+v)/2)$. This is not a precise 'plane'-fronted wave due to the dilaton contribution. This is an eEtrernely impressive expression. We guess the metric has a physical application in noncritical string theory.

We also believe that it is worth while to investigate whether there is exact conformal field theories whose low-energy limit corresponding to the multisoliton solutions obtained in the present paper.

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**Note added:** While this paper was refreed, we received the paper [15] which treated the same solution in different coordinates.